

# EFFECTIVE CROSS SECTIONS AND SPATIAL STRUCTURE OF THE HADRONS

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Informations about the spatial structure of parton distribution within the hadron are provided by the ratios between the inclusive cross sections for a pair of jets, two pairs of jets, three pairs of jets..., and so on. It results, however that these ratios depend not only on the spatial distribution but also, and even more, on the multiplicity distribution of the initial partons.

## 1 Motivations and overall description

The multiple production of pairs of jets in the high-energy hadron-hadron collisions provides a way to investigate the spatial, and also non spatial, structures of the hadrons. A particular instance of this investigation is presented here, taking into account the inclusive cross sections integrated over the momentum spectrum.

When the momentum transfer to the pair of jets  $\Delta p$  is large enough, *i.e.*  $1/\Delta p \ll R_H$ , where  $R_H$  is the hadron radius a description of the process in term of the impact parameter  $b$  is justified. This possibility simplifies many calculations and is moreover very useful in visualizing the processes one wish to study.

In absence of a well established non perturbative QCD, a lot of models may be proposed, all of them stemming from the original partonic description. Using the impact parameter language a list of features which are related to the more general aspects of the experimental evidences is presented:

The distribution of the partons in the transverse plane ( $b$ -distribution).

The distribution in the fractional longitudinal momenta ( $x$ -distribution).

The correlations between transverse and longitudinal variables of the parton:  $(\vec{b}, x)$  and among different partons:  $(\vec{b}_1, \vec{b}_2), (x_1, x_2)$ .

The multiplicity distributions of the incoming partons.

Other features like spin, color, flavor distributions involve clearly finer experimental analyses.

The starting point for the analysis will be, then, the inclusive cross sections for multiple pair production  $d^k \sigma / dp_1 \dots dp_k$ : integrating these expressions over the relevant kinematical variables we get the integrated inclusive cross sections:

$$\sigma_1 = \langle n \rangle \sigma_H, \dots, \sigma_k = \langle n(n-1) \dots (n-k+1) \rangle \sigma_H, \quad (1)$$

where  $\sigma_H$  is the hard contribution to the inelastic cross section.

In order to connect these general definition with the model analysis that will be presented, an expression for  $\sigma_H$  is given here, in a particularly simple case

$$\begin{aligned} \sigma_H^{IJ} &= \int d^2\beta \int \sum_{n=1}^{\infty} \frac{1}{n!} \Gamma_I(x_1, \vec{b}_1) \dots \Gamma_I(x_n, \vec{b}_n) e^{-\int \Gamma_I(x, \vec{b}) dx d^2b} \\ &\times \sum_{m=1}^{\infty} \frac{1}{m!} \Gamma_J(x'_1, \vec{b}'_1 - \vec{\beta}) \dots \Gamma_J(x'_m, \vec{b}'_m - \vec{\beta}) e^{-\int \Gamma_J(x', \vec{b}') dx' d^2b'} \\ &\times \left[ 1 - \prod_{i=1}^n \prod_{j=1}^m (1 - \hat{\sigma}_{ij}) \right] dx_1 d^2b_1 \dots dx_n d^2b_n dx'_1 d^2b'_1 \dots dx'_m d^2b'_m \quad (2) \end{aligned}$$

In this first model the distribution of the partons in the hadron is Poissonian and completely uncorrelated, different distributions will be considered in the following. The integration over the fractional momenta must present a lower bound in order that the parton scattering may involve a finite momentum transfer, larger than a fixed threshold.

The expression in square parentheses of Eq.(2) represents the probability of having at least one semi-hard partonic interaction between hadron  $I$  and hadron  $J$ , the expression  $\hat{\sigma}_{ij} \equiv \hat{\sigma}_{ij}(\vec{b}_i - \vec{b}_j; x_i, x_j)$  is the probability of having a semi-hard interaction of the parton  $i$  from hadron  $I$  with the parton  $j$  of the hadron  $J$ , so it depends on  $x_i, x_j$ , and on the difference of the transverse relative distance  $\vec{b}_i - \vec{b}_j$ , according to the considerations made at the beginning the expression will be taken as local in  $\vec{b}$ :  $\hat{\sigma} = \bar{\sigma}_{x,x'} \delta(\vec{b} - \vec{b}')$ . The cross section results from the sum over all possible partonic configurations of the two hadrons followed by the integration on the overall hadronic impact parameter  $\beta$ .

One will notice that in Eq.(2) all possible interactions between partons of hadron  $I$  and partons of hadron  $J$  are taken into account, so that also all possible hard elastic rescatterings are included.

A relevant simplification is obtained by neglecting every rescattering, *i.e.* by saying that a given parton will interact only once[1]. In that case the expression in the square parentheses in eq.(2) is simplified to:

$$1 - \prod_{i=1}^n \prod_{j=1}^m (1 - \hat{\sigma}_{ij}) \approx \sum_{i,j} \hat{\sigma}_{ij} - \frac{1}{2} \sum_{\substack{i \neq i' \\ j \neq j'}} \hat{\sigma}_{ij} \hat{\sigma}_{i'j'} + \frac{1}{3!} \sum_{\substack{i \neq i' \neq i'' \\ j \neq j' \neq j''}} \hat{\sigma}_{ij} \hat{\sigma}_{i'j'} \hat{\sigma}_{i''j''} - \dots \quad (3)$$

In the same way the inclusive cross section for production of  $k$  pairs of jets with momentum transfer  $p_1, \dots, p_k$  is given by

$$\begin{aligned}
d^k \sigma / dp_1 \dots dp_k &= \int d^2 \beta \int \sum_{n=1}^{\infty} \frac{1}{n!} \binom{n}{k} \Gamma_I(x_1, \vec{b}_1) \dots \Gamma_I(x_n, \vec{b}_n) e^{-\int \Gamma_I(x, \vec{b}) dx d^2 b} \\
&\times \sum_{m=1}^{\infty} \frac{1}{m!} \binom{m}{k} \Gamma_J(x'_1, \vec{b}'_1 - \vec{\beta}) \dots \Gamma_J(x'_m, \vec{b}'_m - \vec{\beta}) e^{-\int \Gamma_J(x', \vec{b}') dx' d^2 b'} \\
&\times k! \quad d\hat{\sigma} / dp_1 \dots d\hat{\sigma} / dp_k dx_{k+1} d^2 b_{k+1} \dots dx_n d^2 b_n dx'_{k+1} d^2 b'_{k+1} \dots dx'_m d^2 b'_m, \quad (4)
\end{aligned}$$

*i.e.*  $k$  partons from the hadron  $I$ ,  $k$  partons from the hadron  $J$  are chosen and connected in all the  $k!$  ways with the elementary cross sections  $d\hat{\sigma}/dp$  the remaining variables are integrated without any constraint. A further integration over the kinematical variables  $p_1, \dots, p_k$  gives the integrated inclusive cross section, in this particular case:

$$\sigma_k = \frac{1}{k!} \int d^2 \beta \left[ \int \Gamma_I(\vec{b}, x) \bar{\sigma}_{x, x'} \Gamma_J(\vec{b} - \vec{\beta}, x') d^2 b dx dx' \right]^k. \quad (5)$$

The effective cross section is introduced in the usual way[2]:

$$\sigma_{\text{eff}} = \frac{\sigma_1^2}{2\sigma_2} \quad (6)$$

and the generalizations for higher integrated inclusive cross sections may be defined in term of dimensionless parameters  $\tau_k$

$$\sigma_k = \frac{(\sigma_1)^k}{k! (\sigma_{\text{eff}})^{k-1} \tau_k}. \quad (7)$$

In this simplified treatment it is clear that  $\sigma_{\text{eff}}$  is mainly connected with the geometrical properties of the hadron, in fact if  $\bar{\sigma}$  is multiplied by a constant then  $\sigma_{\text{eff}}$  remains unaffected, this property holds also for the parameters  $\tau_k$ .

The relevance of the effective cross section  $\sigma_{\text{eff}}$  has been discussed in another talk[3], here the attention is concentrated on the parameters  $\tau_k$ .

## 2 Examples

The population of partons, whichever may be its detailed shape, certainly increases with decreasing  $x$ . When the total energy is so high that hard scatterings can occur even between low- $x$  partons, these processes are more likely than those involving the few valence quarks. This suggests a further simplification obtained by performing an integration in  $x$  of the distribution, assuming

that the kinematical constraints over the  $x$  variables are not very relevant precisely because the small- $x$  processes are the dominant ones. The expression in eq.(5) is substituted<sup>\*a</sup> with:

$$\sigma_k = \frac{1}{k!} \int d^2\beta \left[ \int \bar{\Gamma}_I(\vec{b}) \bar{\sigma} \bar{\Gamma}_J(\vec{b} - \vec{\beta}) d^2b \right]^k. \quad (8)$$

It is now easy to proceed with the actual calculation choosing some definite forms for  $\Gamma$ . Two choices, easy to treat and different enough to allow a first exploration, are:

$$\Gamma_G = \rho \frac{1}{\pi R^2} \exp[-b^2/R^2] \quad , \quad \Gamma_D = \rho \frac{1}{\pi R^2} \theta(R^2 - b^2). \quad (9)$$

In terms of these choices the corresponding values of  $\tau_3$ ,  $\tau_4$  are computed.

The parton population that has been considered till now completely lacks correlations among the partons.

A simple but efficient way of introducing correlation, that allows also a model interpretation is to build up the parton populations in terms of two clusters, having their centers spread over the hadron size<sup>\*b</sup>. To be definite a term of this distribution is written as:

$$\frac{1}{n'!n''!} \int d^2B' d^2B'' f(B') f(B'') \Gamma(\vec{b}'_1 - B') \dots \Gamma(\vec{b}'_{n'} - B') \Gamma(\vec{b}''_1 - B'') \dots \Gamma(\vec{b}''_{n''} - B'') \quad (10)$$

with  $\int d^2B f(B) = 1$ . In so doing the Poissonian character of the integrated distributions is preserved but correlations in the impact parameter are introduced. The actual calculation is performed by choosing:

$$\Gamma = \rho \frac{1}{\pi r^2} \exp[-(\vec{b} - \vec{B})^2/r^2] \quad , \quad f = \frac{1}{\pi R^2} \exp[-B^2/R^2] \quad (11)$$

One verifies that correlations, in form of dependence on  $\vec{b}' - \vec{b}''$  are introduced by the integration over  $B$ . The values of  $\tau_3$ ,  $\tau_4$  are explicitly computed, they depend on the ratio  $u = (R/r)^2$ .

A definite way of departing from the Poisson distribution is to change the original weights of the multiplicities, the general term of the non correlated distribution:

$$\mathcal{N}(C_j) \frac{C_n}{n!} \Gamma(\vec{b}_1) \dots \Gamma(\vec{b}_n) \quad (12)$$

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<sup>a</sup>The expression  $\bar{\Gamma}_I$  represents the effect of an integration in  $dx$ , the "bar" will be omitted in the following.

<sup>b</sup>This description has some similarities with the valon model of R.C.Hwa[4]; however the main attention is directed there to the longitudinal variables, here to the transverse variables.

requires some manageable choice of the coefficients  $C_j$ , in particular an explicit form of  $\mathcal{N}(C_j)$  is needed. A possible choice is a negative binomial distribution for the initial partons<sup>\*c</sup>; it gives for the coefficients and for the normalization term:

$$C_n = (\nu)_n \equiv \nu(\nu+1) \cdots (\nu+n-1) \quad , \quad \mathcal{N}(C_j) = \left[1 - \int \Gamma(\vec{b}) d^2 b\right]^\nu \quad (13)$$

The Poisson distribution is reached in the limit  $\rho \rightarrow \rho/\nu$  and then  $\nu \rightarrow \infty$ . In this way it is possible to measure how much the results deviate from the previous ones when the distribution deviates from the Poissonian form. The shape in  $\vec{b}$  of the parton distribution enters in a way which is independent of the choice of the coefficients  $C_n$ , so different choices, *e.g.* the ones of eq. (11), are possible. The calculation of the quantities  $\tau$  is a bit more laborious than in the Poissonian case, anyhow it can be carried out explicitly.

### 3 Numerical results and conclusions

The numerical results for the quantities  $\tau$  are presented in Table 1.

Table 1: Values of  $\tau$  for different parton populations

	$A_1$	$A_2$	$B$	$C$
$\tau_3$	$\frac{3}{4}$	0.80	$\frac{3}{4} \cdot F_3(u)$	$\frac{3}{4} \left[ \frac{\nu+1}{\nu+2} \right]^2$
$\tau_4$	$\frac{1}{2}$	0.56	$\frac{1}{2} \cdot F_4(u)$	$\frac{1}{2} \left[ \frac{(\nu+1)^2}{(\nu+2)(\nu+3)} \right]^2$

The column  $A_1$  corresponds to an uncorrelated Poissonian distribution and Gaussian shape  $\Gamma_G$  in eq.(9).

The column  $A_2$  corresponds to an uncorrelated Poissonian distribution and rigid disk shape  $\Gamma_D$  in eq.(9).

The column  $B$  corresponds to a Poissonian distribution with correlations and Gaussian shape, eqs. (10,11).

The column  $C$  correspond to an uncorrelated negative binomial distribution and Gaussian shape  $\Gamma_G$ , eqs. (12,13).

The functions  $F_3, F_4$  which appear in column  $B$  are rational functions of  $u$ ; both are equal to 1 when  $u = 0$  and when  $u \rightarrow \infty$ , moreover it results

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<sup>c</sup>This kind of distribution was proposed, in a different context[5] a long time ago.

$F_3(1) = 1.09$ ,  $F_4(1) = 0.93$ , it may be verified in general that they do not vary very much; for this reason the more natural case with three clusters[4] was not worked out. The square parentheses in column  $C$ , which are always less than 1, may differ strongly from unity for small values of  $\nu$ , *i.e.* for distributions that differ much from the Poissonian.

From this preliminary analysis it results that the higher order integrated inclusive cross sections feel, obviously, all the characteristics of the parton distribution, but they are mainly affected by the multiplicity distribution of the incoming partons and less by the spatial shape or by the spatial correlations of the parton distribution.

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